CHROMOSCALE

Unique Language for Sounds Colors and Numbers

Themes:

Base 7, The Harmonic Base
Chromatic Numbers in Base 7
Unique Language in Chromatic Scale
Chromatic Numbers between 0 to 10
Additive and Subtractive Sounds and Colors
Sound Color Number Wheels
Complementary Sounds Colors and Numbers
Chromatic Geometry
CHROMOSCALE - Cyclic Order of Chromotones
Chromotone Fractions
Circle of Fifths
CHROMOSCALE Harmonics and Full MIDI Note Range
Sound & Color APP

Attached files
Hexagonal Lattice
and
Colour Ramping for Data Visualisation
Written by Prof. Paul Bourke

CHROMOSCALE and CHROMOTONES ©
Sound & Color and Base 7 ©
Paolo Di Pasquale
Lighting Designer
Rome, Italy
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The Harmonic Base

The Base 7 is composed from the numbers 0, 1, 2, 3, 4, 5, 6

Table from 1 to 100 in BASE 7

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Six numbers, six times present while the 0 is present only in the external perimeter

This is the harmonic equilibrium of Base 7

(*)

for the differences with Base 10, please visit www.baseseven.com
“exploration between Base 7 and Base 10”

Numerical Table of Base 7 associating numbers with the seven colors of the rainbow.
Chromatic Numbers in Base 7

Hexagonal Table of Base 7 associating numbers with colors.
System where the numbers will be chromatic with the Base 7 characteristics and harmonies.

The hexagon has many interesting properties, and the result is a polygonal table constructible with elementary geometry.

We obtain a shape of hexagram similar to the Koch Snowflake

for this association,
please visit “Texture and Colour”
Prof. Paul Bourke web pages
http://paulbourke.net/texture_colour/

about Hexagonal Table in Base 7
see also attached files, page 14 - 18

“Hexagonal Lattice”
Written by Prof. Paul Bourke
Unique Language in “Chromatic Scale”

It should be emphasized that to pass from one harmonic NOTE to the next harmonic of the same NOTE is enough to double the frequency in Hz. Theoretically, if we double the frequency of the INFRARED we meet on frequency of the ULTRAVIOLET. Based on this principle, we proceed multiplying by 2 the frequency in Hertz of a TONE until you get close to values in GHz of the visible spectrum. In fact, if we multiply by $2^{40}$ the frequency of 369,994 Hz of F# we obtain 406.813 GHz, frequency that corresponds at the border-line of the INFRARED.

We proceed with this system for all the TONES that make up the “Chromatic Scale” by completing the following Table:

### SOUND COLOR CORRESPONDENCE CHART

<table>
<thead>
<tr>
<th>TONE</th>
<th>2^40 Frequency (GHz)</th>
<th>Corresponding Color</th>
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<td>FA#</td>
<td>369.994 Hz x 2^40 = 406.813 GHz</td>
<td>Red</td>
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<tr>
<td>SOL</td>
<td>391.995 Hz x 2^40 = 431.003 GHz</td>
<td>Orange</td>
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<td>SOL#</td>
<td>415.304 Hz x 2^40 = 456.632 GHz</td>
<td>Yellow</td>
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<tr>
<td>LA A#</td>
<td>440.000 Hz x 2^40 = 483.785 GHz</td>
<td>Green</td>
</tr>
<tr>
<td>LA# A#</td>
<td>466.163 Hz x 2^40 = 512.552 GHz</td>
<td>Cyan</td>
</tr>
<tr>
<td>SI B</td>
<td>493.883 Hz x 2^40 = 543.030 GHz</td>
<td>Blue</td>
</tr>
<tr>
<td>DO C</td>
<td>523.251 Hz x 2^40 = 575.320 GHz</td>
<td>Violet</td>
</tr>
<tr>
<td>DO# C#</td>
<td>554.365 Hz x 2^40 = 609.531 GHz</td>
<td>Purple</td>
</tr>
<tr>
<td>RE D</td>
<td>587.329 Hz x 2^40 = 645.775 GHz</td>
<td>Pink</td>
</tr>
<tr>
<td>RE# D#</td>
<td>622.253 Hz x 2^40 = 684.175 GHz</td>
<td>Magenta</td>
</tr>
<tr>
<td>MI E</td>
<td>659.255 Hz x 2^40 = 724.888 GHz</td>
<td>Brown</td>
</tr>
<tr>
<td>FA F</td>
<td>699.456 Hz x 2^40 = 767.961 GHz</td>
<td>Black</td>
</tr>
<tr>
<td>FA# F#</td>
<td>739.988 Hz x 2^40 = 813.626 GHz</td>
<td>Gray</td>
</tr>
</tbody>
</table>

### ELECTROMAGNETIC SPECTRUM

Correspondence between SOUND - COLOR - NUMBER

**BASE SEVEN CALCULATOR**
Paolo Di Pasquale 1988-2012
One Harmonic of “Equal Tempered Scale” is composed by 12 Tones.

Number 12 of Base 10 correspond at number 15 in Base 7

First Harmonic is from 0 to 10
The corresponding numbers of Tones in first harmonic are between 0 and 10

One tone is 10/15
10/15 = 0,40404040...

(F#0 = 0 and F#1 = 10)

0,00000000
F#
0,00000000 + 0,40404040  =   0,40404040  G
1,11111111 + 0,40404040 =  1,51515151  A
2,22222222 + 0,40404040 =  2,62626262  B
3,33333333 + 0,40404040 =  4,04040404  C#
4,44444444 + 0,40404040 =  5,15151515  D#
5,55555555 + 0,40404040 =  6,26262626  F

6,26262626 + 0,40404040 = 10,00000000  F#
(1 octave up)

BASE 7 - Chromatic Numbers between 0 and 10
10/15 = 0,40404040...

(in Base 10  7/12 = 0,58333333....)

about RGB, CMY, HSL and Colour Ramping for Data Visualisation
see attached files, page 19 - 20
Written by Prof. Paul Bourke
Primary colors are sets of colors that can be combined to make a useful range of colors. For human applications, three primary colors are usually used, since human color vision is trichromatic. Additive color primaries are the secondary subtractive colors, or vice versa.

Primary colors are not a fundamental property of light but are related to the physiological response of the eye to light. Fundamentally, light is a continuous spectrum of the wavelengths that can be detected by the human eye, an infinite-dimensional stimulus space. However, the human eye normally contains only three types of color receptors, called cone cells. Each color receptor responds to different ranges of the color spectrum. Humans and other species with three such types of color receptors are known as trichromats. These species respond to the light stimulus via a three-dimensional sensation, which generally can be modeled as a mixture of three primary colors.

Before the nature of colorimetry and visual physiology were well understood, scientists such as Thomas Young, James Clark Maxwell, and Hermann von Helmholtz expressed various opinions about what should be the three primary colors to describe the three primary color sensations of the eye. Young originally proposed red, green, and violet, and Maxwell changed violet to blue; Helmholtz proposed "a slightly purplish red, a vegetation-green, slightly yellowish (wave-length about 5600 tenth-metres), and an ultramarine-blue (about 4820)."

http://en.wikipedia.org/wiki/Primary_color

Correspondence of Sounds, Colors and Numbers regarding Primary Additive and Primary Subtractive

<table>
<thead>
<tr>
<th>PRIMARY ADDITIVE</th>
<th>PRIMARY SUBTRACTIVE</th>
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<tbody>
<tr>
<td>G# RED 1,1111111</td>
<td>G# MGEN 0,0000000</td>
</tr>
<tr>
<td>C GREEN 3,3333333</td>
<td>A# YELLO 2,2222222</td>
</tr>
<tr>
<td>E BLUE 5,5555555</td>
<td>D CYAN 4,4444444</td>
</tr>
</tbody>
</table>

Primary ADDITIVE Sounds = G#, C, E
Primary SUBTRACTIVE Sounds = D, F#, A#
A wheel with primary and secondary colors is traditional in the science as in the arts. Sir Isaac Newton developed the first circular diagram of colors around 1704. Since then, any color wheel which presents a logically arranged sequence of pure hues has merit.

Before the first circular diagram of colors, in 1558 Gioseffo Zarlino in his book “Le Istitioni Harmoniche” draws the first wheel of sounds titled “Numeri sonori”

This is the Sound Color Wheel in Base 7
In color theory, two colors are called **complementary** if, when mixed in proper proportion, they produce a neutral color (grey, white, or black).

In roughly-percentual color models, the neutral colors (grey, white, or black) lie around a central axis. For example, in the HSV color space, complementary colors (as defined in HSV) lie opposite each other on any horizontal cross-section.

Thus, in CIE 1931 color space a color of a particular dominant wavelength can be mixed with a particular amount of the “complementary” wavelength to produce a neutral color (grey or white)

In the RGB color model (and derived models such as HSV)
Primary colors and secondary colors are paired in this way

RED – CYAN
GREEN – MAGENTA
BLUE – YELLOW

Anyway we can apply the same theory of colors either for sounds and Base 7 numbers

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**Complementary**
Colors and Sounds that are opposite each other on the wheel are considered to be complementary

**Analogous**
Analogous scheme use Colors and Sounds that are next to each other on the wheel

**Triad**
A triadic scheme uses Colors and Sounds that are evenly spaced around the wheel
**Chromatic Geometry**

The wheel of sounds in “equal tempered scale” is representable by an hypocycloid curve with twelve cusps. In geometry a hypocycloid is a special plane curve generated by the trace of a fixed point on a small circle that rolls within a large circle. The red curve is the hypocycloid traced as the smaller black circle rolls around inside the larger blue circle. 

(parameters are \( R = 12, r = 1 \) and so \( k = 12 \))

The sine wave or sinusoid is a mathematical curve that described a smooth repetitive oscillation. The sine wave is important in physics because it retains its waveshape when added to another sine wave of the same frequency and arbitrary phase and magnitude. It is the only periodic wave form that has this property. This wave pattern occurs often in nature, including, sound waves, light waves and ocean waves.

To the human ear, a sound that is made up of more than one sine wave will either sound “noisy” or will have detectable harmonics. This may be described as a different timbre. ([http://en.wikipedia.org/wiki/Sine_wave](http://en.wikipedia.org/wiki/Sine_wave))

The correspondence of Sounds, Colors and Base 7 Numbers respects these mathematical and physical principles.

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**BASE 7 - CHROMATIC FRACTIONS**

in “EQUAL TEMPERED SCALE”

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Chromatic Numbers between 0 and 10

0,00000000 = 0

0,00000000 + 0,40404040 = 0,40404040 = 0,40404040 = 1,11111111

1,11111111 + 0,40404040 = 1,51515151

2,22222222 + 0,40404040 = 2,62626262

3,33333333 + 0,40404040 = 4,04040404

4,44444444 + 0,40404040 = 5,15151515

5,55555555 + 0,40404040 = 6,26262626

(1 octave up)
Cyclic Number is an integer in which cyclic permutation of the digits are successive multiples of the number.
Every proper multiple of a cyclic number (that is, a multiple having the same number of digits) is a rotation.

In Base 10 the first “prime number” that produce cyclic numbers is the 7
\( b = 10, \ p = 7 \)  the cyclic number is 0,142857142857....

A cyclic order is a way to arrange a set of objects in a circle.
Set with a “Cyclic Order” is called a cyclically ordered set or simply a cycle.

**Monotone functions**

The "cyclic order = arranging in a circle" idea works because any subset of a cycle is itself a cycle.
In order to use this idea to impose cyclic orders on sets that are not actually subsets of the unit circle in the plane, it is necessary to consider functions between sets.

A function between two cyclically ordered sets, \( f: X \to Y \), is called a monotonic function or a homomorphism if it pulls back the ordering on \( Y \): whenever \([f(a), f(b), f(c)]\), one has \([a, b, c]\).

Equivalently, \( f \) is monotone if whenever \([a, b, c]\) and \(f(a), f(b), f(c)\) are all distinct, then \([f(a), f(b), f(c)]\).

Based on this principle, born the **CHROMOSCALE** in 66 CHROMOTONES

**BASE 7 - CYCLIC CHROMOTONE NUMBERS**

\[
\frac{10}{(10^2-1)} = 0,10101010
\]

| 0,000000 | 1,010101 | 2,020202 | 3,030303 | 4,040404 | 5,050505 | 6,060606 |
| 0,101010 | 1,111111 | 2,121212 | 3,131313 | 4,141414 | 5,151515 | 6,161616 |
| 0,202020 | 1,212121 | 2,222222 | 3,232323 | 4,242424 | 5,252525 | 6,262626 |
| 0,303030 | 1,313131 | 2,323232 | 3,333333 | 4,343434 | 5,353535 | 6,363636 |
| 0,404040 | 1,414141 | 2,424242 | 3,434343 | 4,444444 | 5,454545 | 6,464646 |
| 0,505050 | 1,515151 | 2,525252 | 3,535353 | 4,545454 | 5,555555 | 6,566666 |
| 0,606060 | 1,616161 | 2,626262 | 3,636363 | 4,646464 | 5,656565 | 6,666666 |

The first known occurrence of explicitly infinite sets is in Galileo's last book Two New Sciences.
Galileo argues that the set of squares is the same size as \( S = \{1,4,9,16,25,\ldots\} \) is the same size as \( \mathbb{N} = \{1,2,3,4,5,\ldots\} \) because there is a one-to-one correspondence: \( 1\leftrightarrow 1, \ 2\leftrightarrow 4, \ 3\leftrightarrow 9, \ 4\leftrightarrow 16, \ 5\leftrightarrow 25, \ldots \).
And yet, as he says, \( S \) is a proper subset of \( \mathbb{N} \) and \( S \) even gets less dense as the numbers get larger.
CHROMOSCALE with its 66 Chromotones give the possibility to develop the musical composition with harmonic microtonality and just intonation.
The **Circle of Fifths** is a sequence of pitches or key tonalities, represented as a circle, in which the next pitch is found seven semitones higher than the last.

The Sound Color Wheel on the left shows the Base 7 Cyclic Numbers, while on the right the wheel shows the Circle of Fifths in **C Major**.

At the top of Circle the key of **C Major** has non sharp or flats.

Starting from the apex and proceeding clockwise by ascending fifths, we can subtract the value of **B** (2,626262) to obtain the key of **G** that has one sharp.

From the key of **G**, proceeding to subtract 2,626262 to obtain the key of **D** that has two sharps, and so on.

Similarly proceeding counterclockwise from the apex by descending fifths, proceeding to add 2,626262 to obtain the key of **F** that has one flat, proceeding to add 2,626262 to obtain the key of **Bb** that has two flats, and so on.

We can construct geometrically all Circle of Fifths, Fourths and Thirds with simple algebraic formulas. The same rule can be applied by including all the Chromotones of **CHROMOSCALE**.

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**Circle of Fifths** in Equal Tempered Scale

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<th>RE#</th>
<th>LA#</th>
<th>FA#</th>
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<td>G#</td>
<td>D#</td>
<td>A#</td>
<td>F#</td>
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<td>1,1111</td>
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</table>

We can construct geometrically all Circle of Fifths, Fourths and Thirds with simple algebraic formulas. The same rule can be applied by including all the Chromotones of **CHROMOSCALE**.
CHROMOSCALE Harmonics and Full MIDI Note Range

This Calculator converts numbers and related transaction in numeric form, it shows the corresponding value of sounds in Hz., Colors in Nanometers, RGB, CMYK, XYZ, and the device emits the resulting sound.

SOUND & COLOR APP

This Calculator converts numbers and related transaction in numeric form, it shows the corresponding value of sounds in Hz., Colors in Nanometers, RGB, CMYK, XYZ, and the device emits the resulting sound.

SOUND & COLOR CALCULATOR

with Cycle in 7 Harmonics

it is possible to build musical scores using mathematical functions and formulas simply by playing with sound and color bands.

0 to 100 CYCLE OF SOUNDS

0 to 10 = FIRST HARMONIC
10 to 20 = SECOND HARMONIC
20 to 30 = THIRD HARMONIC
30 to 40 = FOURTH HARMONIC
40 to 50 = FIFTH HARMONIC
50 to 60 = SIXTH HARMONIC
60 to 100 = SEVENTH HARMONIC

THE CALCULATOR CAN WORK IN FOUR DIFFERENT MODES

BASE 7 CALCULATOR – Chromatic Numbers
BASE 10 CALCULATOR – Numerical Sequence
COLOR nm. to SOUNDS Converter
SOUND Hz. to COLORS Converter

about SOUND & COLOR APP
please visit
Hexagonal Lattice

Written by Paul Bourke
December 1997, Updated February 2004

C libraries: hexlib.h, and hexlib.c

Of the three 2 dimensional shapes (equilateral triangle, rectangle, and hexagon) that can be used to tile the plane without holes, the hexagon is the most complex and has many interesting properties. In what follows, an indexing system will be described for a hexagonal tiling called a Spiral Honeycomb Mosaic (SHM). A SHM consists of groups of $n^{th}$ hexagons ($n > 0$) called super-hexagons. It uses a base 7 numbering system for the hexagonal mesh, this is illustrated below for $n = 1, 2,$ and 3.

Software

An application has been developed to explore operations in SHM space. It is based upon X-Windows and OpenGL and is currently available for Mac OS-X and Linux (by request). Note that before running the Mac OS-X version check that X-Windows is running. Download:

macosx.tar.gz

Interface

The interface is straightforward, to find out about any command line options type "hex -h". The left mouse button pans the image, the middle button rolls, and the right button presents a list of menu options. In order to position/rotate/scale the hexagon to the image see the image mapping dialog box. To experiment with SHM operations see the transformation dialog box.

>hex -h
Usage: hex [options]
Options
    -h     this text [help]
    -i a   load input TGA file
    -v     verbose mode
Key commands:
    -, +   zoom in/out
    x      reset
    w      write TGA image of window contents
    1...6  set resolution
    ESC    quit
Mouse buttons:
    left   translate
    middle rotate
    right  menus

Addition and Multiplication

The two basic arithmetic operations can be defined for the SMH, addition and multiplication. These operations act on the addresses of the SMH and result in translation (addition) and rotation/scaling (multiplication) when applied to images represented with the "pixels" of the SMH. The following two C snippets (HexAdd() and HexMul()) implement addition and multiplication. An important characteristics of the operations defined in this way is they are bijective, that is, it is a one to one mapping of each address. Every address (hexagonal pixel) maps uniquely to another address so that no information is lost.
Hexagonal Lattice

Examples

- **This is an example** of repeatedly multiplying an image on an order 5 SHM by 10$_{\text{hex}}$ or 7$_{\text{dec}}$
- **This is the same example** at two powers of 7 greater resolution showing the "superduck".
- **This example** shows repeated addition by 6666$_{\text{hex}}$.

Notes - Spirals

The curve through powers of 7 (10$_{\text{hex}}$) is an equiangular spiral described by

\[ r = a \exp(b \theta) \]

Where \( r \) is the radius and \( \theta \) the angle to the x axis. For example, for the spiral through 1$_{\text{hex}}$, 10$_{\text{hex}}$, 100$_{\text{hex}}$, 1000$_{\text{hex}}$, ... the parameters \( a \) and \( b \) are

\[
\begin{align*}
    a &= \sqrt{3} \\
    b &= \log_7(\sqrt(7)) / \arctan(\sqrt(3)/2) = 1.3632084.
\end{align*}
\]

Since the angle in the above case is taken from the negative y (imaginary) axis, the curve would be traced by

\[
\begin{align*}
    x &= -r \sin(\theta) \\
    y &= -r \cos(\theta)
\end{align*}
\]

The angle between each successive multiples of 10$_{\text{hex}}$ is $\arctan(\sqrt(3)/2) = 40.893395^0$, the ratio of two successive radii is $\sqrt(7) = 2.6457513$.

Online SHM Calculator

**Base Conversion**

- 0
  - [Base 7](#)
  - [Base 10](#)

**Addition/multiplication**

- 0
  - [Base 7](#)
  - [Base 10](#)

Basis 4

Hexagonal Lattice

Other Functions

- Basis 4
- Inverse
- Square root(s)
- Calculate function

Cartesian

- 0
- Base 7
- Base 10
- \(\sqrt{3}\)
- 1.0

References

Alexander, D. and Sheridan, P.

Sheridan, P.
Spiral Architecture for Machine Vision

Sheridan, P., Alexander, D.M.
Invariant transformations on a space-variant hexagonal grid

Sheridan, P., Hintz, T., Alexander, D.
Pseudo-invariant image transformations on a hexagonal lattice
Colour Ramping for Data Visualisation

Written by Paul Bourke
July 1996
Contribution: Ramp.cs by Russell Plume in DotNet C#.

This note introduces the most commonly used colour ramps for mapping colours onto a range of scalar values as is often required in data visualisation. The colour space will be based upon the RGB system.

Colour

The most commonly used colour ramp is often refer to as the "hot- to-cold" colour ramp. Blue is chosen for the low values, green for middle values, and red for the high as these seem "intuitive" bounds. One could ramp between these points on the colour cube but this involves moving diagonally across the faces of the cube. Instead we add the colours cyan and yellow so that the colour ramp only moves along the edges of the colour cube from blue to red. This not only makes the mapping easier and faster but introduces more colour variation. The following illustrates the path on the colour cube.

The colour ramp is shown below along with the transition values.

Again there is a linear relationship of the scalar value with colour within each of the 4 colour bands. In some applications the variable being represented with the colour map is circular in nature in which case a cyclic colour map is desirable. The above can be simply modified to pass through magenta to yield one of many possible circular colour maps.
RGB and CMY

A colour space is a means of uniquely specifying a colour. There are a number of colour spaces in common usage depending on the particular industry and/or application involved. For example as humans we normally determine colour by parameters such as brightness, hue, and colourfulness. On computers it is more common to describe colour by three components, normally red, green, and blue. These are related to the excitation of red, green, and blue phosphors on a computer monitor. Another similar system geared more towards the printing industry uses cyan, magenta, and yellow to specify colour, they are related to the reflectance and absorbance of inks on paper.

HSL, Hue Saturation and Lightness

The HSL colour space has three coordinates: hue, saturation, and lightness (sometimes luminance) respectively, it is sometimes referred to as HLS. The hue is an angle from 0 to 360 degrees, typically 0 is red, 60 degrees yellow, 120 degrees green, 180 degrees cyan, 240 degrees blue, and 300 degrees magenta. Saturation typically ranges from 0 to 1 (sometimes 0 to 100%) and defines how grey the colour is, 0 indicates grey and 1 is the pure primary colour. Lightness is intuitively what it's name indicates, varying the lightness reduces the values of the primary colours while keeping them in the same ratio. If the colour space is represented by disks of varying lightness then the hue and saturation are the equivalent to polar coordinates (r,theta) of any point in the plane.

http://paulbourke.net/texture_colour/colourspace/